

New agegraphic dark energy model with generalized uncertainty principle

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Abstract

We investigate the new agegraphic dark energy models with generalized uncertainty principle (GUP). It turns out that although the GUP affects the early universe, it does not change the current and future dark energy-dominated universe significantly. Furthermore, this model could describe the matter-dominated universe in the past only when the parameter n is chosen to be $n > n_c$, where the critical value determined to be $n_c = 2.799531478$.

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1 Introduction

Observations of supernova type Ia suggest that our universe is accelerating [1, 2, 3, 4]. Considering the Λ CDM model [5, 6, 7, 8, 9, 10], the dark energy and cold dark matter contribute $\Omega_{\Lambda}^{\text{ob}} \simeq 0.74$ and $\Omega_{\text{CDM}}^{\text{ob}} \simeq 0.22$ to the critical density of the present universe. Recently, the combination of WMAP3 and Supernova Legacy Survey data shows a significant constraint on the equation of state (EOS) for the dark energy, $w_{\text{ob}} = -0.97^{+0.07}_{-0.09}$ in a flat universe [11, 12].

Although there exist a number of dark energy models [13], the two promising candidates are the cosmological constant and the quintessence scenario [14]. In order to resolve the cosmological constant problem, we may need to introduce a dynamical, cosmological constant model. The EOS for the latter is determined dynamically by the scalar or tachyon.

Also there exist dynamical models of the dark energy which satisfy the holographic principle. One is the holographic dark energy model [15] and the other is the agegraphic dark energy model [16]. The first is based on the Bekenstein-Hawking energy bound $E_{\Lambda} \leq E_{\text{BH}}$ with the energy E_{BH} of a universe-sized black hole which produces $L^3 \rho_{\Lambda} \leq m_{\text{p}}^2 L$ [17, 18] with the length scale L (IR cutoff) of the universe and the Planck mass m_{p} . The latter is based on the Károlyházy relation of δt [19, 20, 21, 22] and the time-energy uncertainty of $\Delta E \sim t^{-1}$ in the Minkowski spacetime with a given time scale t , which gives $\rho_{\text{q}} \sim \frac{\Delta E}{(\delta t)^3} \sim \frac{m_{\text{p}}^2}{t^2}$. We note that this expression of energy density first appeared in Ref.[21]. Hence we find the vacuum energy density $\rho_{\Lambda} = 3c^2 m_{\text{p}}^2 / L^2$ as the holographic dark energy density [23, 24], whereas the energy density of metric perturbations $\rho_{\text{q}} = 3n^2 m_{\text{p}}^2 / T^2$ with the age of the universe $T = \int_0^t dt'$ as the agegraphic dark energy density. Here the undetermined parameters c and n are introduced to describe the appropriate dark energy model. It seems that the agegraphic dark energy model does not suffer the causality problem of the holographic dark energy model because the agegraphic dark energy model do not use the future event horizon [25, 26, 27, 28, 29]. However, this model suffers from the contradiction to describe the matter-dominated universe in the far past. Hence, the new agegraphic dark energy model with the conformal time $\eta = \int_0^t dt' / a(t')$ with the scale factor $a' = a(t')$ was introduced to resolve this issue [30, 31, 32].

Nowadays we are interested in the generalized uncertainty principle (GUP) and its consequences [33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45] since the Heisenberg uncertainty principle is not expected to be satisfied when quantum gravitational effects become important. Even though the GUP has its origins in the string theory [46, 47, 48], the GUP provides the minimal length scale, the Planck scale l_{p} and may play a role of

evolution of the universe. Especially, we expect that this may modify the evolution of early universe at the Planck scale and inflation significantly.

In this Letter, we investigate the new agegraphic dark energy models with the GUP. We compare this with new agegraphic dark energy models. Especially, we show that the parameter n of the new agegraphic dark energy model with the GUP is restricted to $n > n_c$ for $q = 1$, in order to describe the matter-dominated universe in the far past. As far as we know, this is the first time to incorporate the GUP into the cosmology to explain the dark energy-dominated universe.

2 New agegraphic dark energy model with GUP

We start with extending the time-energy uncertainty to the GUP [33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45]

$$\Delta E \Delta t \geq 1 + \alpha (\Delta E)^2 \quad (1)$$

in the units of $c = \hbar = k_B = 1$. Here the parameter α has the Planck length scale like $\alpha \sim l_p^2 \sim 1/m_p^2$. Solving the saturation of the GUP leads to

$$\Delta E_G = \frac{1}{\Delta t} + \frac{\alpha}{(\Delta t)^3} = \frac{1}{t} + \frac{\alpha}{t^3}, \quad (2)$$

where we use the relation of $\Delta t \sim t$ for cosmological purpose. Then the energy density inspired by the GUP is defined by

$$\rho_G = \frac{\Delta E_G}{(\delta t)^3} \quad (3)$$

where δt is given by the Károlyházy relation of time fluctuations as $\delta t = t_p^{2/3} t^{1/3}$ [19, 22]. For the labelling of α and t_p as

$$\alpha = \left(\frac{q}{n}\right)^2 \frac{1}{m_p^2}, \quad t_p^2 = \frac{1}{3n^2 m_p^2}, \quad (4)$$

respectively, the energy density is described with two parameters (n, q) as

$$\rho_G = \frac{3n^2 m_p^2}{t^2} + \frac{3q^2}{t^4}. \quad (5)$$

The new agegraphic dark energy model is described by using the conformal time η instead of the age of universe T

$$\eta = \int_0^a \frac{da'}{(a')^2 H'} = \int_{-\infty}^x \frac{dx'}{a' H'} \quad (6)$$

with $x = \ln a$ and the Hubble parameter $H' = \dot{a}'/a'$. The corresponding energy density takes the form

$$\rho_G = \frac{3n^2 m_p^2}{\eta^2} + \frac{3q^2}{\eta^4}. \quad (7)$$

A flat universe composed of ρ_G and the cold dark matter ρ_m is governed by the first Friedmann equation

$$H^2 = \frac{1}{3m_p^2}(\rho_G + \rho_m) \quad (8)$$

and their continuity equations

$$\dot{\rho}_G + 3H(\rho_G + p_G) = 0, \quad (9)$$

$$\dot{\rho}_m + 3H\rho_m = 0. \quad (10)$$

The latter determines its evolution with the zero pressure $p_m = 0$ as

$$\rho_m = \frac{\rho_{m0}}{a^3}. \quad (11)$$

Introducing the density parameters $\Omega_i = \rho_i/3m_p^2 H^2$, which implies that the Friedmann equation (8) can be rewritten as

$$\Omega_G + \Omega_m = 1, \quad (12)$$

then one finds

$$\Omega_G = \frac{n^2}{(H\eta)^2} \left[1 + \left(\frac{q^2}{n^2} \right) \frac{1}{m_p^2 \eta^2} \right]. \quad (13)$$

The pressure is determined by Eq.(9) solely, using $dx = Hdt$ as

$$p_G = -\frac{1}{3} \frac{d\rho_G}{dx} - \rho_G \quad (14)$$

which provides the EOS

$$\omega_G = \frac{p_G}{\rho_G} = -1 + \frac{2e^{-x}\sqrt{\Omega_G}}{3n} \frac{\left[1 + 2\left(\frac{q^2}{n^2} \right) \frac{1}{m_p^2 \eta^2} \right]}{\left[1 + \left(\frac{q^2}{n^2} \right) \frac{1}{m_p^2 \eta^2} \right]^{3/2}}. \quad (15)$$

In order to determine ω_G , we obtain the evolution equation from the derivative of Eq.(12) with respect to t together with Eqs.(9) and (10)

$$\frac{d\Omega_G}{dx} = -3\omega_G \Omega_G (1 - \Omega_G) \quad (16)$$

and the relation of conformal time

$$\frac{d\eta}{dx} = \frac{1}{H_0} \left[\frac{e^{-x}}{H/H_0} \right] = \frac{1}{H_0} \sqrt{\frac{e^x(1 - \Omega_G)}{\Omega_{m0}}}. \quad (17)$$

Here we introduce the present Hubble parameter H_0 for our purpose. For numerical computation, we rewrite Eqs.(15) and (17) by introducing $\zeta = H_0\eta$ as

$$\omega_G = -1 + \frac{2e^{-x}\sqrt{\Omega_G}}{3n} \frac{\left[1 + 2\left(\frac{H_0}{m_p}\right)^2 \left(\frac{q^2}{n^2}\right) \frac{1}{\zeta^2}\right]}{\left[1 + \left(\frac{H_0}{m_p}\right)^2 \left(\frac{q^2}{n^2}\right) \frac{1}{\zeta^2}\right]^{3/2}} \quad (18)$$

and

$$\frac{d\zeta}{dx} = \sqrt{\frac{e^x(1 - \Omega_G)}{\Omega_{m0}}}. \quad (19)$$

The EOS of Eq.(18) could be approximated as

$$\omega_G \simeq -1 + \frac{2e^{-x}\sqrt{\Omega_G}}{3n} \left[1 + \frac{1}{2}\left(\frac{H_0}{m_p}\right)^2 \left(\frac{q^2}{n^2}\right) \frac{1}{\zeta^2}\right] \quad (20)$$

for $(H_0/m_p)^2 \ll 1$. Actually, one has $(H_0/m_p)^2 = 3.03 \times 10^{-122}h^2$ with $h \simeq 0.74$. This is just the ratio of the energy density at present and Planck time $\rho_0/\rho_p = (l_p/l_\Lambda)^2 = (H_0/m_p)^2$, which reminds us that the cosmological constant problem arises if one introduces the cosmological constant Λ which satisfies $\omega_\Lambda = -1 = \text{const}$ [49]. That is, observations needs to have $(l_p/l_\Lambda)^2 \leq 10^{-122}$, requiring enormous fine-tuning of the cosmological constant from 1 [50, 51]. In this work, this fine-tuning is naturally included as a correction of the GUP in the EOS. This is because we use a dynamical cosmological constant model of new agegraphic dark energy model with the GUP.

Hence we expect that for the present and future dark energy-dominated universe, the EOS is reduced to that of new agegraphic dark energy model as [30, 31, 32, 52]

$$\omega_G \rightarrow -1 + \frac{2e^{-x}\sqrt{\Omega_G}}{3n} = w_n. \quad (21)$$

Considering Eq.(21) together with the condition $a \rightarrow 0$ ($x \rightarrow -\infty$) of the far past, the matter-dominated universe is recovered with $\omega_n = -2/3$ and $\Omega_n = n^2a^2/4$, while the radiation-dominated universe is recovered with $\omega_n = -1/3$ and $\Omega_n = n^2a^2$ [30]. However, this prediction comes from the EOS ω_n only. We remind the reader that the pictures of far past and far future should be determined from Eq. (16) which governs the whole evolution of the new agegraphic dark energy model [52]

In the case of new agegraphic dark energy model with the GUP, one finds from Fig. 1 that the whole evolution depends on parameter n critically for $q = 1$. Here the initial condition is given by $\Omega_{G0} = 0.72$ and $\eta_0 = 1/H_0(\zeta_0 = 1)$ at the present universe. If n is less than the critical value $n_c \sim 2.799531478$, then its far past behavior is not acceptable because of $w_G \rightarrow \infty$ and $\Omega_G \rightarrow 1$ as $x \rightarrow -\infty$. On the other hand, if n is greater than the

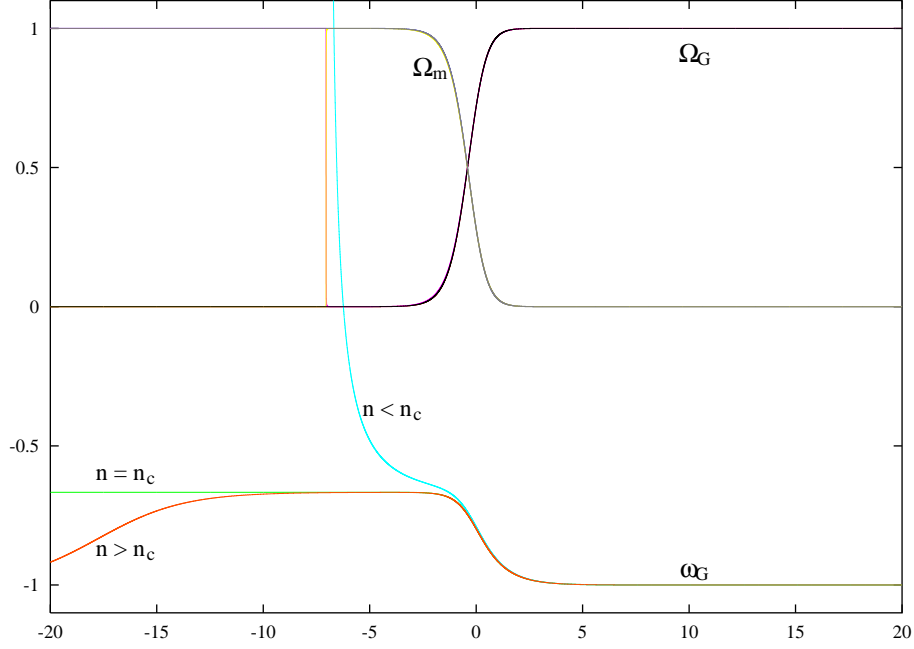


Figure 1: Graphs for the evolution of the new agegraphic dark energy based on the GUP for $q = 1$: x vs Ω_i , ω_G . From top to down, the density parameters $\Omega_G(\Omega_m)$ and the EOS ω_G are depicted for $n < n_c$ ($\omega_G \rightarrow \infty$), $n = n_c = 2.799531478$ ($\omega_G \rightarrow -2/3$) and $n > n_c$ ($\omega_G \rightarrow -1$), respectively.

critical value, then $w_G \rightarrow -1$ and $\Omega_G \rightarrow 0$ as $x \rightarrow -\infty$. In this case, we expect to have $\Omega_G \propto a^2 = e^{2x}$ from w_G in Eq.(18). If n approaches the critical value, then $w_G \rightarrow -2/3$ and $\Omega_G \rightarrow 0$ as $x \rightarrow -\infty$. This corresponds to the matter-dominated universe in the far past, predicted by Wei and Cai [30]. However, in the far future we have the convergent results of $\Omega_G \rightarrow 1$, $\omega_G \rightarrow -1$, irrespective of n . This behavior is the same as the new agegraphic dark energy models did show [52].

3 Discussions

We discuss the effects of the GUP on the new agegraphic dark energy models. The GUP is relevant to the Planck time, $t = t_p = 10^{-43}s$. The GUP explains the cosmological constant problem very well because it implies the Planck scale, $l = l_p$. Actually, the GUP does not change the present and future dark energy-dominated universe significantly. In order to get the Planck time behavior, the simulation must be being performed to arrive $x = -120$ from $x = 0$. However, this task is formidable to us and thus we did not see what

happens in this limit. Our simulation was performed for the finite range of $x \in [-20, 20]$ only.

In conclusion, the new agegraphic dark energy model with the GUP induces the Planck scale in the evolution of universe. However, the GUP does not modify the present dark-energy dominated universe significantly.

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